Miners and Modals

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This paper extends the miners problem from Kolondy and MacFarlane 2010 to epistemic 'should's. I show these cases clarify the nature of the problem: in particular, non-probabilistic extensions of the classic semantics for 'should' fail to account for them. I show that a hybrid possible worlds/probabilistic account does better and suggest that miners cases are good evidence that our semantics for 'should' and conditionals can access probabilistic information.

1. The Classic Semantics for 'Should' The classic semantics for 'should' is as follows:

(1) $[\![\textit{should } \phi]\!]^{c,w,f,g} = 1$ iff $\forall w' \in BEST(w,f,g) : [\![\phi]\!]^{c,w,f,g} = 1$

where $BEST(f,g,w) = \{w \in \bigcap f(w) : \neg\exists w' : w' <_{g,w} w\}$ and $w' \leq_{g,w} w''$ iff $\{p \in g(w) : w' \in p\} \supseteq \{p \in g(w) : w'' \in p\}$ (Kratzer 2012, von Fintel 2012). This semantics is usually paired with the Kratzer 2012 for conditionals (where $f + \phi$ is the function f' st. $\forall w : f'(w) = f(w) \cup \{\phi\}$):

$[\![\textit{if } \phi \textit{ then should } \psi]\!]^{c,w,f,g} = 1$ iff $[\![\textit{should } \psi]\!]^{c,w,f+\phi,g} = 1$.

2. Information Sensitivity: As Kolodny and MacFarlane 2010, Charlow 2011 and Cariani et al. 2013 have established, the following kind of case is problematic for this semantics:

Miners. Ten miners are trapped either in shaft A or in shaft B, but we do not know which. We can block one shaft, but not both. If we block the right shaft we save all the miners, but if we block the wrong shaft we kill all the miners. If we block neither shaft, just one miner will be killed.

Here the following sentences are true:

(1) I should block neither shaft.
(2) If the miners are in shaft A, I should to block shaft A.
(3) If the miners are in shaft B, I should to block shaft B.

To predict all of the above are true, a semantics for 'should' must be *information sensitive*. Where g is an ordering source, f_1, f_2 are modal bases, $BEST$ is information sensitive iff for some w, f_1, f_2, $f_1(w) \subseteq f_2(w)$, $w \in BEST(f_2, g, w)$ but not $w \in BEST(f_1, g, w)$. Since the classic semantics is *not* information sensitive, it cannot predict that (1)-(3) are simultaneously true.

3. Epistemic cases: So far, the literature has presupposed that the miners problem is limited to deontic 'should'. The following, I claim, is an example where 'should' has an epistemic reading:

Exam. Alex and Billy are the top math students in their class and will take their weekly algebra exam tomorrow. The following is true:

- Alex does best in 66% of the exams.
- Given that Billy studies tonight, Billy will probably get the best grade: out of exams he studied for, Alex did best in 66% of them

- Given that Billy doesn't study, Billy will certainly not do best. Alex did better in all of the exams that Billy didn't study for.
- Billy always lets a fair coin toss decide whether he will study. He studies just in case it comes up heads.

When 'should' is read epistemically, the following sentences are true here:

(4) Alex should do best.
(5) But if Billy studied, then he should do best.

Given the set up, the set of best worlds should *not* entail that Billy studies nor that he doesn't. To predict (4) is true the set of best worlds to entail that Alex does best. $BEST(f,g,w) \subseteq$ *Alex does best* and $BEST(f,g,w) \nsubseteq$ *Billy studies*. To predict (5) is true, we want the set of best worlds which are worlds where Billy studies to be ones where Billy does best. So $BEST(f +$ *Billy studies*$, g, w) \subseteq$ *Billy does best*. But, as in **Miners**, for all these conditions to be met, $BEST$ must be information sensitive.

4. Problems for Conservative Solutions: Conservative responses tackle the miners problem using purely possible worlds semantics. One option, outlined in von Fintel 2012 is to posit a context shift from a subjective to an objective deontic "should" here, thus rejecting the assumption that (1) - (3) are evaluated in the one context and avoiding any modifications to the classic semantics. But positing ordering sources analogous to the subjective/objective distinction overgeneralises in the epistemic case: we falsely predict that in **Exam** there can be true readings of sentences like:

(6) If Billy studied, then Alex should do best

Another prominent conservative account, that of Cariani et al. 2013, grants that there is information sensitivity and uses a set of propositions representing the decision problem faced by agents in cases like **Miners** to get an ordering on worlds which is information sensitive. But there is no obvious way to reinterpret CKK's decision parameter in such a way to get the right predictions here. This highlights a more general problem facing conservative accounts: in cases like **Exam** there are no analogues for the deontic parameters that get us the right results.

5. Solution: I claim information dependence is a probabilistic phenomenon and show how we might account for it within a hybrid probabilistic possible worlds framework. We retain the semantic entry in (1) and the Kratzer account of conditionals but give a different definition of $BEST$. Firstly, we depart from the classic semantics in using orderings on propositions to generate an ordering on worlds. We say that g, the ordering source, is a function from worlds and modal bases to orderings on propositions $\precsim_{f,g,w}$. From an ordering $\precsim_{f,g,w}$, we can first define a set of best propositions:

$$PBEST(f,g,w) = \{p | \neg\exists q : q \precsim_{f,g,w} p \text{ and } \neg p \precsim_{f,g,w} q\}$$

This allows us to define a domain of quantification for 'should':

$$BEST(f,g,w) = \bigcup\{S : S \text{ is a maximal consistent intersection of elements of } PBEST(f,g,w)\}$$

where A is a maximal consistent intersection of elements of B iff there's no $C \in B$ s.t. $A \cap C \subset A$ and $\neq \varnothing$.

To account for deontic miners cases, we suppose that $g(w)$ yields an ordering which tracks the relevant expected utilities:

$p \precsim_{w,f,g} q$ iff $EU(p|\bigcap f(w)) \geq EU(q|\bigcap f(w))$

Given probabilities and utilities consistent with the case, the unique proposition in $PBEST(f, g, w)$ will entail that I block neither shaft; this makes (1) true. However, when we restrict the ordering to just the worlds where the miners are in shaft A, this generates a new ordering based on the expected utilities conditional on the fact that the miners are in shaft A. In this case, the unique best proposition entails that I block A; this makes (2) true. Analogous reasoning holds for (3).

To account for epistemic cases, we can use an ordering which orders proposition based on whether they meet a threshold probability or not:

If $P(p|\bigcap f(w)) \geq t$, then, for every q, $p \precsim_{f,g,w} q$.
If not, then $p \precsim_{f,g,w} q$ iff $P(p|\bigcap f(w)) \geq P(q|\bigcap f(w))$.

Given probabilities consistent with the case, we get the result that some best proposition entails that Alex will do best; this predicts the truth of (4). When we update the ordering with the modal base restricted to the worlds where Billy studies, some best proposition entails that Billy will do best. This predicts the truth of (5).

6. Upshot: Epistemic miners cases help us clarify the nature of the problem. They strongly suggest that information sensitivity is genuinely needed and that it is a probabilistic and not a deontic phenomenon. Giving a semantics access to probabilistic information yields a general solution to the problem. I suggest that information sensitivity is good evidence that the compositional machinery of weak necessity modals and conditionals can be sensitive to probabilistic information.

References: Cariani et al. 2013, "Deliberative Modality under Epistemic Uncertainty"; Charlow 2011, "What We Know and What to Do"; von Fintel 2012, "The Best We Can (Expect to) Get?"; Kolodny and MacFarlane 2010, "Ifs and Oughts"; Kratzer 1991, "Modality"; Kratzer 2012, "The Notional Category of Modality"; Kratzer 2012, "Conditionals".